Problem Solving Model Developed By Pre-Service Mathematics Teachers

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The main aim of the study was to investigate the problem solving models used by 50 pre-service mathematics teachers. The pre-service teachers were divided into 10 groups of 5 and then asked to solve three non-routine mathematical problems, one problem per week for three weeks. The groups were encouraged to write all the steps they used in solving each problem. They were observed as they solved the problems by two of the researchers. The study used document analysis in the form of writtenresponses from pre-service teachers. Their solutions to the problems were analysed in relation to George Polya's problem solving model. The pre-service teachers' responses revealed the models they used to solve the problems. The steps used by pre-service teachers in solving the problems turned out to be the generic problem solving models. Pre-service teachers' use of a model in the problem solving process enabled them to achieve deeper levels of understanding in order to adequately describe the solution process.

Key words: *Problem Solving Model, Pre-Service Teachers, Non-Routine Problems, Problem Solvers*

DOI : 10.5281/zenodo.17280305

Introduction

Problem solving plays an important role in developing learners' mathematical thinking (Carson, 2007), which results in an advanced transition from operational to structural outlook (Maharaj, 2007). Such thinking skills should be developed by focusing on how to think rather than what to think (Wilson, 1993). The South African education policies both at the school and at teacher education level encourage that approaches to teaching should involve problem solving. The Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education, 2011a) places emphasis on teaching mathematics using problem solving for primary and secondary schools. The teacher education policy which is called Minimum Requirements for Teacher Education Qualifications (MRTEQ) (Department of Higher Education, 2015) stipulates that problem solving should be one of the exit level outcomes for teacher education programmes.

One of the solution strategies which have been found to be useful to learners when dealing with problem solving in mathematics is the use of models (Bao, 2016; Sepeng & Kunene, 2015). Models in problem solving play a major part in making students understand the problem. Bao (2016)posits that the use of models in problem solving affords problem solvers an opportunity to interpret the problem through a drawing, visually represent the problem situation and relevant relationship as well as to choose appropriate mathematical operations and procedures. Research on the use of models and other strategies in teaching learners with disabilities indicate that learners performed better when taught problem solving using models (Morin, Watson, Hester & Raven, 2017; (Xin et al., 2011). Gaigher, Rogan, and Braun (2006)) recommend that students should be given an opportunity to construct mental models, and also learn to apply their models. The model that seems to be the basis in problem solving is Polya's problem-solving model. This model has a sequence of four steps namely; understand a problem, develop a plan to solve it, execute the developed plan, and verify the obtained solution(Polya, 1945). Recent studies have further highlighted the role of emotions and motivation in problem solving. For instance, Stuppan et al. (2025) showed that STEM-based problem-solving tasks can trigger learners' epistemic curiosity, which enhances engagement and perseverance in solving complex problems.

The aim of education worldwide is to teach students and learners to be good problem solvers, not only in the classroom but in life as well. Such individuals will be successful in life (Temel, 2014). The criticism levelled against South African education at tertiary level is that the popular teaching method is lecturing and students actively take notes whether written or said orally (Loji, 2012). This method does not strengthen the student teachers' knowledge of mathematics. Learners who are in future going to be taught by these student teachers will

also have weak knowledge of mathematics which results in learners' poor performance in mathematics. This assertion is supported by the research conducted by Mumthaz and Kgomotso (2016)which reveals that there is a relationship between the extent to which prospective teachers have been prepared and their impact on the learner achievement.

Research Problem

Problem solving has been extensively researched in schools. There is however, a paucity of such research at the Initial Teacher Education (ITE) level. This study, therefore investigates problem solving models developed by pre-service mathematics teachers in solving non-routine mathematical problems. In this paper, the terms pre-service mathematics teachers and mathematics student teachers will be used interchangeably.

The aim of the study was to determine the processesused by pre-service mathematics teachers to solve non-routine mathematical problems.

The research question is: How do pre-service mathematics teachers solve non-routine mathematical problems?

Literature Survey

This article focuses on problem solving from the cooperative, socio-cultural and constructivist perspectives of learning. The researchers looked at the problem solving executed in a group setting, positive peer relationship and effective social skills are therefore developed. Also, individual differences are understood and respected by team members (Niewoudt, 2014; (Amosun, 2016)). Student teachers individually construct knowledge while in a group setting.

Problem solving in mathematics refers to a mathematical situation that poses a mathematical problem, but there is no obvious mathematical solution (Dostál, 2015; Nieuwoudt, 2015). Problem solving also involves understanding the problem so as to decide on the information needed for a solution to solving the problem and evaluating the appropriateness of the solution (Ozturk & Guven, 2016)

The concept problem solving varies from routine problem solving (Mogari & Lupahla, 2013) which (Nieuwoudt, 2015) calls the traditional approach to the non-routine problem solving. In the case of routine problem solving steps or procedures that a student has done previously when solving other problems are used(Yazgan, 2013). On the other hand the non-routine problem solving requires the application of various thinking skills and strategies which one of them is modelling the problem (Elia, van den Heuvel-Panhuizen, & Kolovou, 2009). The case study conducted by (Mabilangan, Limjap, & Belecina, 2012)amongst five secondary school students revealed that the strategy of "Make a Model or Diagram" was the most frequently used problem solving strategy by both proficient and apprentice problem solvers. This shows that modelling in problem solving enables students to better understand and develop an insight on how to solve the problem. More recent research also supports deliberate practice combined with structured problem-solving frameworks. Miller et al. (2025) found that deliberate practice in undergraduate STEM courses helped align student approaches with expert-like reasoning, making problem solving more effective than repetitive procedures alone.

Studies indicate that there is a relationship between problem solving and metacognition (Ozturk & Guven, 2016; Du toit & Du toit, 2013; Wilson & Clark, 2004). It has been found that good problem solvers are better users of metacognition during problem solving. They state that problem solving activities enhance learner metacognition, because learners who are good problem solvers are usually also self-aware thinkers. Problem solving also depends on the perception of an individual. individuals with positive perception of their problem solving abilities are usually better problem solvers as compared to those with negative perception of their problem solving abilities(Kilpatrick, Swafford, & Findell, 2001a).

Methodology

Sample Selection

Participants were purposively selected from the final year pre-service teachers in the Bachelor of Education (Mathematics, Science and Technology Education) Programme. Only those taking mathematics education in their final year participated in this study. The rationale for this selection was that these pre-service teachers would provide rich information and deep insights about the models which they could use during problem solving at their work places. These participants had completed five semesters of mathematics content, two semesters of the subject pedagogy as well as teaching practicum. Consequently, they are familiar with the South African school curriculum and the demands it places on the use of models in problem solving in the teaching and learning of mathematics.Beyond traditional curriculum requirements, innovative frameworks such as the "Ways of Thinking in Engineering Design-based Physics" (WoT4EDP) highlight the importance of scaffolding metacognition and integrating computational thinking into problem-solving tasks (Subramaniam et al., 2025). Such approaches could complement initial teacher education practices.

Data Collection

Fifty participants were divided into ten groups with five members each. They were given three non-routine mathematical problemsto solve, one problem per week for three weeks. The groups were encouraged to write all

the steps they used in solving each problem. Data was also collected through interviewing a focus group which was selected from the ten groups. This particular group was selected because of its response to problem 2 where they provided three different solutions. They were chosen due to their relevance to the current South African mathematics curriculum and their suitability to the phase in which these teachers would be practicing.

Analytical Process and Unit of Analysis

The purpose of this study was to find which processes were used by pre-service mathematics teachers when solving non-routine mathematical problems. The units of analysis were the written solutions provided by pre-service mathematics teachers. This demanded that pre-service teachers' responses be chunked so that each can reveal the strategies used to solve the problem.Chunks that supported understanding the problem, devising a plan, executing the plan and looking back(Polya, 1945) we developed. The purpose was neither to evaluate the responses in relation to the completeness or appropriateness of the method used nor the correctness or wrongness of the responses but to identify the processesbeing used to solve the problem. Datafrom individual groups were analysed and compared so as to establish thethinking processes used and to formulate the model categories per given non-routine mathematical problem. The formulated group models were then compared to find the general problem solving model developed by this cohort of pre-service mathematics teachers. The data collected from the interview was analysed in order to validate our interpretation of the solution process and consequently the model developed by pre-service mathematics teachers.

Data Analysis

Problem 1

Emily has her model train set up on a circular track. There are six telephone poles, evenly spaced around the track. It takes the engine of the train 12 sec to go from the first to the third pole. At this same rate, how long will it take the engine to go completely around the track?

In an attempt to solve this problem, the process used by members of **Group A** was read the question, sketch a representation of the problem, and then solve the problem (see their sketch in figure 1). Figure1 indicates that they misinterpreted the problem because their train track is linear instead of circular as demanded by the problem. Their misrepresentation of their sketches (Figure 1) made them fail to get the correct answer. They needed to have six spaces in between the six poles and yet their sketches allowed them five only.There was no attempt to check if the answer was correct.

Figure 1: Group A's drawing for Problem 1

Figure 3: Group J's drawing for Problem 1

Figure 4: Group G's drawing for Problem 1

Group B used the same process as **Group A** and equally failed to represent the information on their sketch (Figure 2). The concept of circular track was problematic to them. In their rectangular allay of linear blocks they allowed three spaces each equal to 12s from first pole to fourth pole instead of two spaces each equal to 12s from first pole to third pole, thereby getting the wrong answer of 24 seconds. Both Groups **A and B** failed to construct proper relationships between the given quantities (poles, length of track segment, and time).

The third group to get the wrong result was **Group J**. They used the same process of reading the problem, interpreting, sketching and solving. Their sketch in figure 3 initially used 6 blocks. The six blocks were then divided into 2 groups, each one representing the given 12 seconds, obviously not producing the correct answer. The idea of 'circular' finally dawned and they produced the second sketch in figure 3 but they failed to interpret the gaps and hence failed to apportion the 12 seconds.The major problem was that they grouped the poles into two groups of three and hence have two sets of 12 seconds. This ignored the concept of space between poles as required by the problem.

The process used by members of **Group C**was read, interpret, sketch a representation of the problem, and then find ananswer to the problem (Figure 5). The rest of the **GroupsD to I**followed the same process, though with different sketches and varying explanations. **Group G**(figure 4) used logical deduction over and above the sketch. In their problem-solving process there is no evidence that they attempted to do some reflection on the solution.

Problem 2

Al is holding four cards in his hand: the ace of spades, the ace of heart, the ace of clubs, and the ace of diamonds. Steve pulls two of the cards from Al's hand without looking. What is the probability that Steve has pulled at least one black ace?

Group C attempted **Problem** 2 thrice. The initial process used was read, interpret, sketch a representation of the problem, and then find ananswer. On the top left corner of figure 5(a),they tried to interpret the problem by listing all the possible outcomes. On the right side they then tried to sketch their interpretation but they failed to apportion the correct probabilities, and hence they failed to get to a conclusive answer. Actually they got a solution of 1, which they scratched. This forced them to look back at the problem and do another attempt.

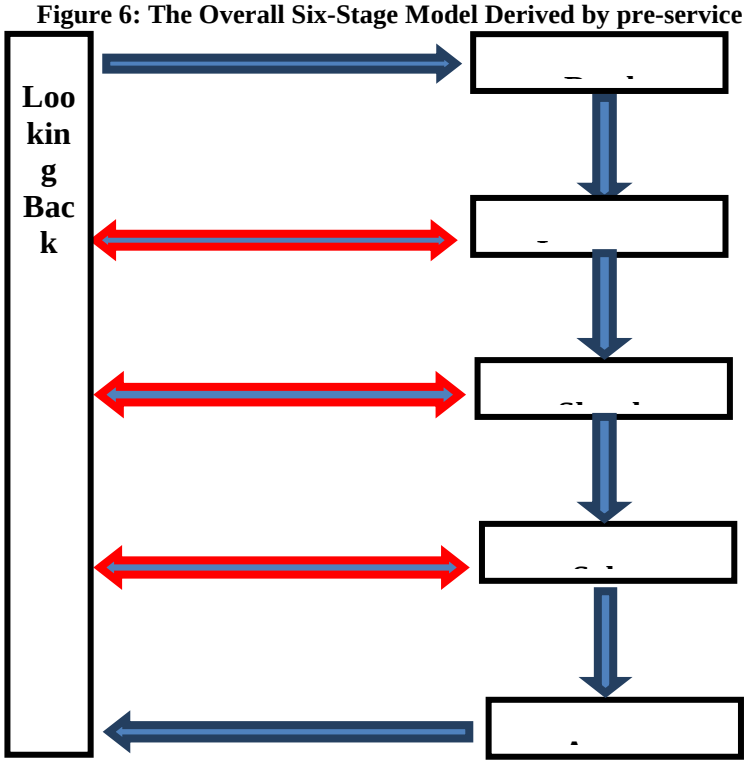
Figure 5(a): Group C's drawing for

Figure 5(b): Group C's drawing for Problem

Figure 5(c): Group C's drawing for Problem 2

Their second attempt is in figure 5(b). In this case they tried to just list all the possible outcomes and count them. Unfortunately, the intended outcome of at **least** one black ace got lost in interpretation again. They did not count an outcome of two black aces and hence got a two thirds solution which they were not happy about.
On the third attempt (figure 5(c)) they got all their parameters right by listing the possible outcomes correctly and hence they got the correct answer. This demonstrated that the group did a lot of reflection at every stage of the problem solving process.
The problem-solvingprocessesof the 10 groups were analysed and put together. The outcome is the six-stage problem solving model in figure 6. In this model all groups of students started by reading the question. They tried to interpret their reading and then sketched a diagram to represent the problem. They then solved the problem and finally came to ananswer. But it was interesting to note that they tried to look back in order to check their correctness at every stage. This is shared by(DeGuire, 1980)who posits that looking back is part of the problem solving process rather than the end of the problem solving process.

Figure 6: The Overall Six-Stage Model Derived by pre-service teachers



Discussion
This study reports on the mathematical problem solving processes used by final year pre-service teachers majoring in mathematics in their final year. The model presented here (figure 6) consolidatesthe processes from the ten groups that took part in the study. The model could best be described as generic since it is similar to those developed by other researchers such as the SOLVED model by (Hohn & Frey, 2002) and the I.D.E.A.L. model by (Herald, 1991) except that it is cyclic in nature compared to existing linear models.
Mathematics teachers need to understand the role of modelling in theteaching and learning of mathematics in general. In this way they will be in a better position to incorporate modelling in their teaching thus enhancing learners' mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001b) while at the same time achieving the aim of the South African School Curriculum which states that "Mathematical modelling is an important focal point of the curriculum" (Department of Basic Education, 2011b, p. 8).
The study seems to have been limited by pre-service teachers' lack of mathematical computational and analytic skills, mathematical practices and weak comprehension of the mathematical problems. For example, some groups when diagrammatizing the train problem misinterpreted 'circular' in the problem to 'linear' as indicated by their diagrams. This is worrying since these teachers are in the final year of their studies. However, the findings are informative and could be used to initiate conversation between lecturers and curriculum developers regarding the inclusion of modelling in all modules offered in the initial teacher education programme.
There is no evidence that most of the groups looked back once they got ananswer except two groups (**C** and **J**). This observation is similar to the findings of (Baki, Kosa, & Guven, 2011; Lee, 2016; Mubark, Zaman, & Mubark, 2012) who found that looking back was absent in learners' work. An emerging concern in recent years is the reliance of students on generative AI tools to bypass reflective stages of problem solving. Wang et al. (2024) caution that while

such tools can scaffold problem solving, they may also act as a crutch if not embedded within tasks that encourage deeper reasoning.The observation that most pre-service teachers did not reflect on the process once the solution was attained is concerning since there is consensus among researchers in mathematics education that teachers should focus on learner thinking process rather than the correctness or wrongness of the answer. In this study 'looking back' could have been limited by pre-service teachers' past experiences with school mathematics where reflection is often limited to validation of the answer.

Conclusion

The purpose of developing a model is not to teach pre-service teachers to follow fixed rules but to provide them with a tool to support their understanding of the problem so that they can identify the variables and their relationships, problem conditions and mathematical operations necessary for solving the problem. The use of a model building provides a toolwhich can be used to assess the robustness of the problems solvers mathematical knowledge and other mathematical behaviours

Recommendations

The discussion above shows that pre-service teachers should be trained to solve mathematical problems using models and such models should be shared to enrich pre-service teachers' knowledge. The looking back stage, especially after attaining an answer should be given more emphasis during modelling. Pre-service teachers should be exposed to various methods of solvingmathematical problems. Since modelling is one of the goals of teaching in South Africa (Department of Basic Education, 2011b), pre-service teachers should be afforded opportunities during training to develop their modelling skills. This means that concerted effort should be made to significantly include modelling in initial teacher educationprogrammes. This could be emphasised during micro-teaching and teaching practice periods, thus supporting pre-service teachers in developing a culture of modelling.

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