

# The Effectiveness of specific Teaching Strategy for Approximate Roots on the Estimation Skill in Mathematics of the Intermediate First- Year Female Students

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## Abstract

*The present research aims at knowing "the effectiveness of specific teaching strategy for approximate roots on the estimation skill in mathematics of the intermediate first- year female students". The sample research was consisted of (62) female students which splatted to two equivalent groups (experiment, and control) groups. The researcher prepared tool to measure estimate skill in mathematics, consisting of (20) multiple-choice items with three alternatives, she has been verified sincerity. The reliability coefficient is about (0,76). The results are analyzed using, "an equation t-test for two independent samples", as well as that "the efficiency equation" was used for Mac-Gogian . The researcher found that: "there was a statically significant difference at (0.05) level of significance between the means of the (the experimental, and control) groups in the estimation test, due to the experimental group".*

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## 1. Introduction

Mathematics textbooks to this day lack a specific formula for approximating surmised roots (quadratic and cubic). In the books of (History of Mathematics) many different formulas have been dealt with in the accuracy of their results to find the products of square and cubic roots (approximate). I also mentioned methods and lengthy processes to reach the product of approximate roots in books of (calculus), and from one of these followed processes, he mentioned the scientist (Hoffman, 2012) is a linear approximation process that states:

"If the function  $f$  is differentiable at  $x = a$  then the equation of the tangent line to the function  $f$  at the point  $x = a$  takes the following form:

$$f(a) + f'(a) \cdot (x - a) = L(x)"$$

If the point  $x$  is close to the point  $a$ , then the values of the tangent line function close to the values of the function  $f(x)$  and therefore:

$$f(a) + f'(a) \cdot (x - a) \approx f(x)$$

<http://www.amazin.com/contemporary>

It is known that the square root function  $\sqrt{x} = f(x)$  is a derivative function on the field of positive rational numbers, and that the equation of the tangent line for this function at  $x = a$ , will be:

After performing the arithmetic operations, the previous equation becomes as follows:

The value of  $x$  near to the attachment point  $a$  in which the square root of  $x$  will be nearer from the tangent line equation, and according to the linear approximation process (which we referred to above), in the approximate formula to find the square root of  $x$ , are:

(Where  $a$  point is close to  $\underline{x}$ )

Let  $x = f(x)$ , and let  $x$  be a positive non-square integer, choose the closest square integer to  $x$ , let it be  $b^2$ , so the approximate formula for the square root of  $x$  is in the equation (1) below:

$$\sqrt{x} \approx \frac{b^2 + x}{2b} \quad (1)$$

(Provided  $b^2$  is the closest perfect square to  $x$ )

Formula (1) will be adopted in determining the current search strategy, to find the products of the approximate square roots of positive integers, without resorting to the use of an electronic calculator.

Thus, if we consider the cube root function  $g(x) = x$ , and let  $x$  be a non-cubic integer, we can, in the same manner as above, obtain the approximate formula for the cube root of  $x$ , in equation (2) below:

$$x \approx \frac{x^3}{b^3} \dots \dots \dots (2)$$

(Provided  $b^3$  is the closest complete cube to  $x$ )

Formula (2) will be adopted in determining the current search strategy, to find the products of the approximate cubic roots of the integers, without resorting to the use of an electronic calculator. This makes it easier for the learner to be able to use the skill of appreciation.

The current research is concerned with using a specific strategy based on (the linear approximation process) to teach the topic (estimating approximate roots) and calculating its effectiveness in improving the appreciation skill in mathematics among first-grade intermediate students.

### Research problem:

The American National Council of Mathematics Teachers (NCTM, 2000) emphasized an increase in interest in using approximations in order to solve mathematical problems, and in order to verify the plausibility of the results. On the other hand, the Council recommended to reduce the interest in machine training (the use of paper and pen) in solving problems (Obeid, 2010, 48-51).

Currently in the teaching of first-grade mathematics in our schools, the estimation of the square roots of numbers that do not represent perfect squares, and the estimation of the cubic roots of numbers that do not represent complete cubes, using automatic procedures that do not give the answer closest to the correct one, but give several answers, for example; To estimate the square root of the number (7), we limit the number 7 between two square values (4.9), and because 7 is closer to the number 9 than to the number 4, the approximate answer is: (2.7, 2.8, 2.9 2.8 7), and to estimate the root, then, the cube of the number (30), in which we limit the number 30 between two cubic values (27.64), and because the number 30 is closer to the number 27 than to the number 64, the answer is:

(130 +3.1, 3.2, 3.3) (same math, 2018: 57-59)

In view of the scarcity of previous studies investigating the topic of approximate roots (deafness) in school mathematics at the global and local level, the researcher decided to prepare a specific strategy for teaching the topic of approximate roots (which was mentioned in the introduction to the research) for real numbers and measuring their effectiveness in the skill of estimation.

In light of the above, the researcher raises the following question:

What is the effectiveness of a specific strategy for teaching the topic of approximate roots on the skill of appreciation in mathematics among first-grade intermediate students?

### Research Importance:

The importance of the current research stems from: -

- The importance of the role of appreciation in learning arithmetic, when learners guess their answers before performing their written calculations, they focus on the meaning of the numbers and the operations involved and not on the automatic procedures (Badawi, 2007: 499).

This research may benefit the mathematics curriculum developers for the first intermediate grade to include the content of the specific strategy for teaching the approximate roots (deaf) included in this research.

- This research may benefit mathematics teachers by guiding to the use of an algorithm to obtain a single result that is as close to correct as possible to estimate approximate roots (square and cubic), instead of the automatic method used in the textbook.

### Research objective:

The importance of the current research stems from: -

- The importance of the role of appreciation in learning arithmetic, when learners guess their answers before performing their written calculations, they focus on the meaning of the numbers and the operations involved and not on the automatic procedures (Badawi, 2007: 499).

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- This research may benefit mathematics teachers by guiding to the use of an algorithm to obtain a single result that is as close to correct as possible to estimate approximate roots (square and cubic), instead of the automatic method used in the textbook.

**Research objective:**

The current research aims to identify a specific strategy for teaching the topic of approximate roots (square and cubic), and its effectiveness on the appreciation skill in mathematics of first-grade intermediate students.

**Research hypothesis:**

To achieve the research goal, the following null hypothesis was formulated:

"There is no statistically significant difference at the level of significance (0.05) between the mean scores of the two groups (experimental and control) students in the assessment skill test in mathematics.

**Search Limits :**

The current research is limited to:

- Female students of the first intermediate grade in the middle schools of the General Directorate of Education in Baghdad - Rusafa Al-Aula.
  - The subject of (estimating square and cubic roots) from the second chapter of the mathematics book for the first intermediate grade (first part), 3rd edition, for the year 2018 AD.
  - The first semester of the academic year (2018-2019).
- Approximate roots include (square, cubic).

**Defining Terms:****The Strategy:**

I defined it (Al-Bakri and Al-Kiswani, 2002) as "specific procedures for teaching a specific topic, as it means determining the pattern of actions and behaviors that the teacher will conduct in order to achieve certain results" (Al-Bakri and Al-Kiswani, 2002: 125).

The researcher defines it procedurally as: a set of organized steps by which the researcher trains the students of the experimental group to teach the subject of estimating approximate roots (square and cubic).

**Approximate roots:**

The researcher defines them procedurally as: the roots of rational numbers (do not represent whole squares or cubes), in which the approximate answer is sufficient.

**Skill:**

Khawaldeh(2007) defined it as: "a pattern of behavior that requires the ability to act and coordinate information" (Al-Khawaldeh, 2007: 266).

**- Appreciation:**

Reys (1992) defined it as: "A procedure to obtain an speculative answer to solve a specific problem without the use of arithmetic and without the use of paper and pen" (Al-Absi, 2009: 204). **Appreciation skill:**

The researcher defines it procedurally as: a skill performed by the students of the experimental group to guess the answer closest to the correct one, and it is measured by the students 'grades in the assessment test prepared by the researcher (for this purpose).

**Theoretical background:**

Abu Zina, (1997) defined the teaching strategy as: a set of instructive matters that direct the teacher's work path and his course during the lesson (Abu Zina, 1997: 121).

In light of the procedural definition of the strategy (contained in defining terms), the researcher defines an instructional strategy with organized, guiding steps used to teach the subject of estimating approximate roots from the mathematics book for the first intermediate grade.

**The teaching strategy defined for this research:**

**First: The researcher devised an algorithm to find an approximate square root product for a positive non-square integer based on the linear approximation process.** Refer to the approximate formula in Equation (1) included in the introduction to the research, according to which the strategy includes the following steps:

The teacher teaches to find an approximate result of the deaf square root of a positive integer  $x$ , as follows:

Find the nearest perfect square to  $x$  (greater or less than  $x$ ), say  $b^2$ .

- Add the number  $x$  (to find the square root) to the square number  $b^2$ .

Divide the result of the second step by twice the number  $b$ , and you get an approximate result of the number  $x$ .

The teacher gives multiple examples of the topic, resolved with the participation of students, for example;

Find an approximate answer for the square root of 5:

- You choose the closest square number to the number 5, which is the number 4.

- You add the number 5 to the number 4, so the result is 9.

- Divide the result 9 by twice the number 2, so that the final answer is 2.25, which represents the product of the approximate square root of the number 5.

Note if you compare this result with the actual result using a calculator, which is equal to 2.23, and we find that we get a specified result (single) rounded to one decimal place. The teacher gives practical life issues on the topic, for example; Find an approximate side length for the known square area.

Second: The researcher devised an algorithm to find an approximate result of the cube root for a non-cubic integer based on the linear approximation process. Refer to the approximate formula in Equation (2) included in the introduction to the paper, according to which the strategy includes the following steps:

The teacher teaches to find an approximate result of the deaf cube root of an integer  $x$ , as follows:

You choose the nearest cubic number greater or less than  $x$ , let it be  $b^3$ .

- Add the number  $x$  to twice the cubic number  $b^3$ .

- Three times the square of the number  $b$  is found.

Divide the product of the second step by the product of the third step, and you get an approximate result of the cube root of  $x$ . The teacher gives multiple examples on the topic, and it is solved with the participation of students, for example;

Find an approximate result of the cube root of 10.

You choose the nearest cube number to 10, which is 8.

- You add the number 10 to twice the number 8, and you get the number 26.

- You find three times the square of number 2 which equals 12.

- Divide the number 26 by the number 12, and you get an approximate result of the cube root of 10 is 2.1666.

Compare this result with the real product using a calculator which equals 2.1544, and you will find that you got a specified (single) product rounded to one decimal place.

• The teacher gives practical life issues on the topic, for example; Find an approximate side length of a cube-shaped polygon whose volume is known. Third: To find an approximate result of the square root of a positive non-square  $y/x$  relative number, the researcher concludes the following:

• You find an approximate result of the square root of both the numerator and denominator, by applying the approximate formula (1) to them, and keeping the result in the form of regular fractions.

• Divide the numerator by the denominator, and you get an approximate result of the required number.

As can be seen from the following example;

Find the result of the approximate value of the square root of the rational number (4.4).

• Reframe the relative number (4.4) from the decimal fraction formula into the normal fraction formula  $10/44$ .

• You find the approximate value of the square root of both the numerator and denominator in the form of regular fractions, so you get:

$$\bullet \text{ Dividing the product of 44 by the product of 10, you get } \frac{1976}{9314} \approx \frac{44}{10}$$

You find that you get a rounded answer to three decimal places.

Fourth: The researcher concludes in the same manner as mentioned in (Third) finding an approximate result of the cube root for a non-cubic relative number  $x/y$ .

### **The researcher concludes:**

For real numbers that are in the form of square roots or cubic roots (deaf), one of the four strategies referred to above can be used to find approximate results for them.

### **Appreciation skill in mathematics:**

The scientist (Sowder, 1988) defined the estimation as: "a conceptual network that is precisely organized so that the learner can make connections between numbers and characteristics of operations" (Al-Absi, 2009: 205).

There are three types of estimation associated with mathematics, and they are:

- Estimate answers to verbal questions.

- Estimating the answers of the arithmetic operations

Estimating measurements of some physical objects (Bell, 1987, Part 1: 120).

The second type is dealt with in this paper, with regard to the products of square roots or cubic (approximate) operations.

### **To develop the skill of appreciation among the learners:**

He mentioned (Shoen&Zweng, 1989) that the teacher should consider the following:

- Giving learners examples using number estimation.

- Use examples from learners' practical life.

- Learners are not required to have a high level of accurate answers.

- Giving the learners a chance to speak with each other to discuss the results.

- Presenting examples that solve the same issue in different ways (Nofal and Al-Absi, 2006:

### **211). The difference between rounding and estimating:**

Approximation is finding a result that is accurate enough, but estimation is a mental skill resulting from an educational guess.

Approximation does not require guesswork, and approximation is sometimes used in estimation.

A good estimator possesses high mental arithmetic skills, and the reverse is not true (Reys, 1986: 22-23), (Hall, 1984: 516-517).

The researcher believes that:

Approximation has rules that control it. As for appreciation, it varies from one individual to another according to the individual's experience.

The estimate is related to all types of measurements (lengths, angles, areas, volumes, weights, ..., etc.),

The rounding is usually used to round numbers to the nearest whole number, or to the nearest tenth, or to the nearest hundred, and other numbers.

#### **Previous Studies:**

1- A study (Hernandez & Romero, 2004):

In this study, "Newton's method was developed to approximate (nth-roots) to a real number".

<https://doi.org/10.1080/00207160410001714583>

1- A study (John, 2007):

The study was conducted in Germany, and it aimed to identify the method of finding the approximate square root, and its method is summarized as follows:

Let  $x$  be the square root of the number  $N$ , so:

The first step:

$$N / x = x$$

The second step:

$$(x + N / x) / 2 = x$$

The third step:

If  $y$  is the approximate square root of  $N$ , then:

$$(y + N / y) / 2$$

It is a better approximation

The fourth step:

- If  $x^2 < N$ , then use the lower bound of  $x$ , and choose a number that is the midpoint of the distance between the upper bound and  $x$

- If  $x^2 > N$ , find an upper bound for  $x$ , and choose a number that is the midpoint of the distance between the lower bound and  $x$

Fifth step: Apply step (3).

<http://www.alhoor.world.com//sciehlis76asp>

#### **1- A study (Nofal and Al-Absi, 2006):**

The study was conducted in Jordan, and it aimed to identify "the effect of an educational program - a computerized learning in developing the appreciation skill in mathematics among third-grade basic students." The study sample consisted of (86 individuals (42 individuals for the experimental group, 44 individuals for the control group).

The results of the study showed "a statistically significant difference between the mean scores of the two experimental groups and the control group on the assessment test" attributed to the educational program for the benefit of the experimental group students (Nofal and Al-Absi, 2006: 208).

The researcher's comment: In this research a method was adopted to find the product of real numbers in the form of approximate roots (quadratic or cubic) based on the linear convergence process, and its effectiveness in the estimation skill was measured.

#### **Search procedures:**

The researcher adopted "experimental design with partial control for two equal groups and the post test." Al-Nu'man medium was intentionally chosen as the research sample, and the school contained two divisions for the first intermediate grade, and one of them was randomly chosen to represent the experimental group and the other to represent the control group, and after excluding students who had failed (statistically) so that their previous experiences did not affect the research results, the number of students reached The research sample is (62) students (31 experiments and 31 officers), and the equivalence of the two groups in the variables (chronological age, intelligence, pre-assessment) was verified before conducting the experiment, as shown in the following table (1):

Table (1) T-test results for equivalence variables

| Group             | Experimental group (31) |          | control (31) |          | T-value  |                         |
|-------------------|-------------------------|----------|--------------|----------|----------|-------------------------|
| Variables medium  | medium                  | variance | medium       | variance | computed | tabular                 |
| Pre-estimation    | 9,75                    | 5,085    | 8,86         | 7,612    | *1,671   | 2 via freedom degree 60 |
| Chronological age | 191,40                  | 20,729   | 193,07       | 33,373   | *1,264   |                         |
| Intelligence      | 18,77                   | 35,569   | 17,40        | 85,877   | *0,692   |                         |

\*Not significant at a significance level (0.05)

The shape inference scale was applied (Daniels, 1986) on 11/3/2019, and it is a non-verbal intelligence test consisting of (45) items appropriate to the Iraqi environment, and the indications of validity and reliability were extracted in the study (Al-Dulaimi and Abdullah, 2002).

#### Research tool (assessment test):

The researcher defined the behavioral goals for the content of the topic (approximate roots), then formulated the items of the assessment test that amounted to (20) paragraphs in the initial form of the multiple choice type with three alternatives.

**Validity of the test:** The researcher confirmed the validity of the test after presenting it to a group of experts and arbitrators. Exploratory experiment: The researcher applied the test on an exploratory sample consisting of (50) female students from the average ship for girls, and the average time spent in answering the test was (50) minutes, after which the researcher corrected the test and arranged the students' grades in descending order, and sorted the highest (50%) The values of the difficulty coefficients for the paragraphs ranged between (0.63-0.24), and the values of the discrimination coefficients for the paragraphs ranged between (0.65 -0.31) (and the discriminatory strength of these paragraphs is an indicator of the validity of the construction of the test), and the coefficients were The effectiveness of the alternatives to the wrong test items are all negative, which indicates their effectiveness. Then the stability factor was calculated for the test using the (Cuder-Richardson 20) equation, and the reliability factor was approximately (0.76).

With this, the number of the final assessment test items reached (20) the same paragraph, and the researcher prepared the test instructions, and gave the following example:

Estimate an approximate value of the square root of the number 7 (without using the calculator).

(a) 2.5 (b) 2.6 (c) 2.7

Answer: (b)

Then the researcher applied the assessment test (which she prepared for research purposes) on 1/12/2019 under her supervision.

#### Presentation and interpretation of results:

The validity of the research hypothesis was verified by calculating the arithmetic mean and variance of the grades of the two groups' students (experimental and control), and a difference was observed between the mean scores of the two groups, and to calculate the significance of the difference between them statistically, the equation "T-test for two independent samples" was used, and the results are shown in Table (2). As in the following:

Table (2) the results of the t-test for the two groups of research in the assessment test (post)

| The group    | Arithmetic medium | variance | T - test |                                | statistical significance At 0.05 |
|--------------|-------------------|----------|----------|--------------------------------|----------------------------------|
|              |                   |          | Computed | tabular                        |                                  |
| Experimental | 16.25             | 11.42    | 3.677    | 2 at a degree of freedom of 60 | significance                     |
| Control      | 12.87             | 14.78    |          |                                |                                  |

Table (2) shows that the calculated t-test value is greater than the tabular value at the level of significance (0.05) and the degree of freedom (60), thus rejecting the null hypothesis, because there is a statistically significant difference between the mean scores of the two groups (experimental and control) and in favor of the experimental group.

The researcher also measured the rate of efficacy using the (Mac-Gogian) equation, and the result was as shown in Table (3) as follows:

Table (3) the percentage of effectiveness of the teaching strategy specified on the assessment test

| Group | Test | Arithmetic medium | total score of the test | Percentage of effectiveness (Mac-Gogian). |
|-------|------|-------------------|-------------------------|-------------------------------------------|
| 0.634 | 20   | 16.25             | Post-test               | experimental                              |
|       |      | 9.75              | Pre-test                |                                           |

Table (3) above shows that the rate of effectiveness is acceptable for the teaching strategy specified for the experimental group between the two tests (pre and post) of the skill of appreciation.

The percentage spoken for Mac-Gogian was greater or equal to (60%) (Eid, 2006, 250). And the interpretation of these results may return (according to the researcher's opinion) to:

- The method of presenting the strategy specified for this research, attracted the students' attention and made them happy to find a way to get a single answer to the product of any approximate root, unlike what was happening in the classroom of obtaining several values of the approximate root product of a certain real number.

- As well as because there is a close relationship between approximation and estimation, estimation is a strategy for solving problems, in which the approximate answer is sufficient.

The estimate is used to verify the reasonableness of the calculations that were made with pen and paper (Badawi, 2007: 498).

### Conclusions:

Based on the results of the research, the researcher concludes the following: 1- Teaching, using the strategy defined for the topic of approximate roots, contributed to raising the appreciation skill in mathematics for female students of the first intermediate grade.

2- The effectiveness of the teaching strategy specified for the topic of approximate roots on the skill of appreciation in mathematics among first-grade intermediate students.

### Recommendations and Proposals:

In light of the research findings that have been reached, the following can be drawn up:

- The researcher recommends: Directing the attention of curriculum and textbook authors in the Ministry of Education on including the strategy specified for this research in the subject of estimating approximate roots from the mathematics textbook for the first intermediate grade, for its effectiveness in the skill of estimation.

### The Researcher Proposes:

Conducting a study to devise a strategy corresponding to the strategy specified for this research, to teach the approximate nth root in the field of real numbers and to know its effect on the skill of decision-making.

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