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Conceptual And Procedural Knowledge In Mathematics And Its Relationship To Mathematical Thinking Among Jordanian Teachers

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Abstract

Here we investigate the level of conceptual and procedural knowledge in mathematics and its relation to mathematical thinking among the teachers of grades one to three in Jordan. In this quantitative study, 100 teachers were randomly selected from the Directorate of Education of the Al-Jameah state for the academic year 2018/2019. The researcher tested the conceptual and procedural knowledge of these teachers, as well as their mathematical thinking. The level of conceptual knowledge, procedural knowledge, and mathematical thinking among the teachers were low, and there were significant differences (α = 0.05) when comparing their conceptual and procedural knowledge; procedural knowledge was greater the conceptual knowledge. There was a positive relationship between the level of conceptual knowledge and mathematical thinking, and also a positive relationship between the level of procedural knowledge and mathematical thinking.

1. Introduction

Mathematical knowledge is divided into two main parts: conceptual knowledge and procedural knowledge. Conceptual knowledge is a set of close links between concepts, generalizations, facts, principles, and mathematical laws. Those with conceptual knowledge should be able to present examples of mathematical concepts and express them in drawings, models, and shapes, and realize the complementarity and interdependence between basic concepts and sub-concepts. Regarding procedural knowledge, Rittle-Johanson, Siegler, and Albibali (2001) and Star (2005) see procedural knowledge as specific and sequential procedures and steps to reach a solution to a particular mathematical task using the language of symbols and rules that can be applied.

Some researchers have pointed out that there is a relationship between conceptual knowledge and procedural knowledge. For example, the acquisition of conceptual knowledge is a precedent and necessary for the acquisition of procedural knowledge (Halford, 1993). Some scholars believe that both procedural and conceptual knowledge develop and are formed independently of each other, and this classification was adopted by Baker and Czarnocha(2002). Some authors have proposed that the relationship between conceptual and procedural knowledge is mutual and simultaneous, and that the acquisition of one is simultaneous with the acquisition of the other (Rittle-Johanson et al., 2012).

There have been calls for a greater balance between conceptual and procedural knowledge in teaching. These calls arise from concerns that teachers have become overly focused on procedural knowledge (NCTM, 2000; Kashim, 2016).

Mathematics is an abstract science that results from the creativity of the human mind. Mathematics concerns the methods of thinking, and is based on conclusions and evidence. Jeremy Kilpatrick (2020) and José Antonio Rodríguez-Martínez (2020) defined thinking as "a cognitive mental process that expresses the relationships between things and is a series of mental activities that the human brain performs when exposed to a stimulus in order to obtain a result, decision or solve a problem", which distinguishes man from other animals.

Hence, the aim of this study was to measure conceptual and procedural knowledge in the teaching of mathematics and its relationship to mathematical thinking among the teachers in Jordan (grades one to three).

Hill et al. (2005) reported that there is a close correlation between a teacher‟s mathematical knowledge and student achievement. The decisive factor in the success of the teaching and learning process is the teacher, and the teacher‟s efficiency depends on their knowledge, the material they are studying, and their method for relaying this knowledge (Hill et al., 2005). The conceptual and procedural knowledge of mathematics teachers in primary schools in Riyadh with regard to relativistic preparation was conducted. This study indicated the mathematics teachers in primary schools in Riyadh have not a good conceptual and procedural knowledge regarding relative numbers with significant differences between the average conceptual degrees of teachers and the average degrees of their procedural knowledge (Kashim, 2016). Therefore, the aim of this study was to

measure the level of conceptual and procedural knowledge in the teaching of mathematics and its relationship to mathematical thinking among the teachers.

Methods

The study problem

The researcher found a problem for the teachers of the first three grades in the level of conceptual knowledge and procedural knowledge necessary to teach mathematics, and the degree of their ability to practice mathematical thinking skills. In the Ministry of Education schools as a teacher for the first three grades. Also, this study acquires its importance because we will be able to know the level of conceptual and procedural knowledge needed to teach mathematics and its relationship to mathematical thinking for the teachers of the first three grades. Also, the importance application are:

The classroom teacher preparation program in colleges of education may benefit from the results of this study to work to improve the quality of these programs and reconsider the plan and develop it by the type of mathematics courses prescribed for them, and the number of these courses.

The results of this study may benefit curriculum planners, those who prepare the teacher and train him, and educational policy makers in rebuilding sound decisions that take into account the achievement of student learning in the first degree.

The study problem is determined in answering the following main question: What is the level of conceptual and procedural knowledge necessary for teaching mathematics and its relationship to mathematical thinking for the teachers (grades one to three)?

The main question is sub-divided into the following sub-question.

Research questions

1. What is the level of conceptual knowledge necessary for teaching mathematics among Jordanian teachers (grades one to three)?
2. What is the level of procedural knowledge necessary for teaching mathematics among Jordanian teachers (grades one to three)?
3. What is the level of mathematical thinking among Jordanian teachers (grades one to three)?
4. Is there a significant difference between the average performance of Jordanian mathematics teachers in the conceptual knowledge test and the average performance in the procedural knowledge test?
5. Is there a significant correlation between the level of conceptual knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three)?
6. Is there a significant correlation between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three)?

Study methodology

The researcher used a quantitative descriptive approach to evaluate the level of conceptual and procedural knowledge in the teaching of mathematics and its relationship to mathematical thinking among Jordanian teachers (grades one to three).

Study population and sample

The study community consists from Jordanian teachers (grades one to three) who hold a „class teacher‟ bachelor‟s degree; teach the first grades (first, second and third grades) in Amman Governorate in the Directorate of Education - liwaaaljamah; and were teachers for the 2018–2019 academic year. According to the data obtained from the Directorate of Education-liwaaaljamah, the study sample consisted of 100 female teachers. The schools were randomly selected, and then the study tools were applied to Jordanian teachers (grades one to three) in these schools of the Ministry of Education in the Amman Governorate in the Directorate liwaaaljamah for the year 2018–2019, where the percentage of representation of the sample to study community was 36%.

Measures

The researcher performed the following procedures

The online training course was obtained from the Center for Mathematics Education and Learning at the University of Michigan Learning Mathematics for Teaching (LMT) to obtain tools of mathematical knowledge necessary for teaching mathematics (Mathematical Knowledge for Teaching; MKT), and these tools were translated. The questions representing both conceptual knowledge and procedural knowledge were chosen from mathematical knowledge tools (MKT) and used to measure the levels of conceptual and procedural knowledge needed to teach mathematics. Also, the mathematical thinking tool developed by Al-Fayez (2016) was used to measure the level of mathematical thinking for the teachers of the first three grades. Then, the study tools were presented to the jury committee, and the tool was modified according to the opinions of the committee. Also, the validity and reliability of the study tools were verified.

The study sample was chosen from Jordanian teachers (grades one to three) in the Amman governorate in the academic year 2018–2019, and the study tools that included the conceptual knowledge and the procedural

knowledge from MKT and the mathematical thinking from Al-Fayez (2016) on the study sample were applied. The tools (tests) were corrected, and the data was entered into the SPSS program for statistical analyses.

Approval was obtained from both the Ministry of Education-Jordan and the World Islamic science and education university to apply the study to Jordanian teachers (grades one to three) working within its schools.

Study tools

Conceptual and procedural knowledge tests:

Both conceptual and procedural knowledge tests for numbers and operations for the teachers (grades one to three) depend on the tools of the mathematical knowledge for teaching (MKT) which developed at the Center of Mathematics Education and Learning At the University of Michigan Learning Mathematics for Teaching (LMT).

The conceptual and procedural knowledge test consisted of 15 and 13 questions, respectively: 1 mark for a correct answer and zero for a wrong answer. The validity of the tools was verified by presenting the tools to experts in mathematics, measurement, assessment and evaluation, curricula, and specialists in the English language.

The tools of the conceptual and procedural knowledge test was confirmed by the test-retest method,where the test was applied to an exploratory sample consisting of 30 female teachers (grades one to three)not part of the study sample. The test was re-applied after 2 weeks on the same sample, and the Pearson correlation coefficient for conceptual and procedural knowledge test were 0.87 and 0.86 in the test and re-test, respectively. Also, the stability coefficient was also calculated using the method of internal consistency according to the Kuder-Richardson-20 equation, where the stability valueswere 0.93 and 0.82 (considered appropriate for the purposes of this study). Also, the test was applied to30 female teachers (grades one to three)not part ofthe study sample, and the researcher determined that1 h was required to complete each test.

Mathematical thinking test

The researcher used the mathematical thinking tools prepared by Al-Fayez (2016) to measure the level of mathematical thinking and its suitability for the study target group; this tool included seven theoretical definitions (induction, conclusion, mathematical proof, logical thinking, generalization, modeling, and guessing), where the test included 27 questions distributed among the seven theoretical definitions of mathematical thinking (four questions for each definition).

The Pearson correlation coefficient for mathematical thinking test that developed by Al Fayez (2016) was (0.074) in the test and re-test. The researcher also tested the internal homogeneity by presenting the test to a group of arbitrators with jurisdiction, and their views were taken, and appropriate amendments were made to the test paragraphs, and thus the number of mathematical thinking test paragraphs in its final form became 20 paragraphs, seven essay questions (short answers) and thirteen multiple choice type questions, which are corrected by giving (1) for the correct answer and (zero) for the wrong answer.

The stability of the mathematical thinking test was confirmed by the test-retest method, as the test was applied to an exploratory sample consisting of 30 participants (grades one to three) outside of the study sample, and the test was re-applied after 2 weeks. The test-retest Pearson correlation coefficient was 0.92. The stability coefficient was also calculated using the internal consistency method according to the Koder Richardson formula-20, where the stability value was 0.90, and these values were considered appropriate for the purposes of this study. Based on the pilot study of 30 female teachers, the researcher determined that 1 h was needed to complete the tests.

Results

This study aimed to identify the level of conceptual knowledge and procedural knowledge in mathematics and its relationship to mathematical thinking among Jordanian teachers (grades one to three).

Question 1: What level of conceptual knowledge is required to teach mathematics among Jordanian teachers (grades one to three)?

The arithmetic means for testing the conceptual knowledge necessary to teach mathematics among Jordanian teachers (grades one to three) reached 3.89 (Table 1). Referring to the criterion for classifying levels of conceptual and procedural knowledge, the level of mastery of conceptual knowledge was very low (26%).

Table 1. The arithmetic means, standard deviation, and percentage of the conceptual level of knowledge necessary to teach mathematics among Jordanian teachers (grades one to three).

	Arithmetic mean	standard deviation	level	Percentage
Conceptual Knowledge Test Final Mark (15)	3.89	1.78	Very low	26%

Question 2: What level of procedural knowledge is required to teach mathematics among Jordanian teachers (grades one to three)?

The arithmetic mean for the test of procedural knowledge necessary to teach mathematics among Jordanian teachers (grades one to three) reached 5.25. Referring to the criterion for classifying levels of conceptual and procedural knowledge, the level of mastery of procedural knowledge was very low (40%) (Table 2).

Table 2. The arithmetic mean, standard deviation, and percentage of the level of procedural knowledge necessary for teaching mathematics among Jordanian teachers (grades one to three).

	Arithmetic mean	standard deviation	level	Percentage
Procedural Knowledge Test College Score (13)	5.25	2.204	Very low	40%

Question 3: What is the level of mathematical thinking among Jordanian teachers (grades one to three)?
Table 3 shows that the arithmetic means for testing the level of mathematical thinking among Jordanian teachers (grades one to three) reached 5.74. Referencing the approved criterion, the level of mastery of mathematical thinking skills was less than the approved criterion, where the percentage of mathematical reasoning level reached (27%).

Table 3. The arithmetic mean, the standard deviation, and the percentage of the level of mathematical reasoning among the parameters of the first three grades.

	Arithmetic mean	standard deviation	level	Percentage
Mathematical Thinking Test Overall Score (20)	5.47	2.129	Less than the approved test	27%

Question 4: Is there a significant difference between the average performance of Jordanian mathematics teachers in the conceptual knowledge test and the average performance in the procedural knowledge test?
We used a t-test to test for a difference in the performance of teachers in the conceptual knowledge test and the procedural knowledge test (Table 4). We detected significant differences ($p < 0.05$) between teachers ‟performance on the conceptual knowledge test and their performance on the procedural knowledge test in mathematics teaching; the teachers had better procedural knowledge than conceptual knowledge.
Table 4. Arithmetic averages, standard deviations, and t-test for correlated data of the difference between teachers‟ performance on the conceptual knowledge test and their performance on procedural knowledge test in teaching mathematics.

	Number	Arithmetic mean	Standard deviation	t-test	Degreesoff reedom	Statistical significance
Conceptual Knowledge Test (Overall Score 100)	100	25.93	11.869	-8.663	99	0.000
Procedural Knowledge Test (Overall Score 100)	100	40.38	16.951			

Question 5: Is there a significant correlation between the level of conceptual knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three)?
To answer this question, the Pearson correlation coefficient was calculated between the level of conceptual knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three) (Table 5).

Table 5. Pearson correlation coefficient of the relationship between the level of conceptual knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three).

		Mathematical thinking test
Conceptual knowledge test	Correlation coefficient r	0.323**
	Statistical significance	0.001
	Number of teachers	100

Table 5 shows that there was a positive relationship between the level of conceptual knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three) ($t = 0.323$, $p = 0.001$).
Question 6: Is there a significant correlation between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three)?

To answer this question, the Pearson correlation coefficient was calculated between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three) (Table 6).

Table 6. Pearson correlation coefficient of the relationship between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three).

		Mathematical thinking test
Procedural knowledge test	Correlation coefficient r	0.676**
	Statistical significance	0.000
	Number of teachers	100

We detected a statistically significant positive relationship between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three). ($r = 0.676$, $p = < 0.001$).

Discussion

This study aimed to identify the level of conceptual knowledge and procedural knowledge in mathematics and its relationship to mathematical thinking among Jordanian teachers (grades one to three). We found that Jordanian teachers (grades one to three) have a „very low" mastery (26%) of conceptual knowledge in mathematics. This finding suggests that the teachers tend to memorize information and apply procedures and processes in an automatic and routine manner, without having any deep knowledge of the concepts (Tchoshanov, 2011). This finding might be because of a lack of interest in the academic preparation programs for these teachers. For mathematics education to achieve the desired results, teachers much have a strong conceptual basis in mathematics (Tchoshanov, 2011; Johari Surif, 2020). Rather, based on our data, it seems that teachers simply memorize rules and procedures, and used them as algorithms to solve problems, without understanding the underlying concept (Schneider et al., 2009; Zuya, 2017, Kashim, 2016).

We found that Jordanian teachers (grades one to three) also have „very low" (40%) mastery of procedural knowledge relevant to mathematics teaching. This result might be a result of the nature of the college programs taken by those specializing in teaching, whichare overly simple and superficial, andinclude only two courses in mathematics (Groth & Bergner, 2006). Most college students specializing in teaching come from literary branch secondary schools and lack interest in mathematics (Zuya, 2017, Kashim, 2016).

The Jordanian teachers (grades one to three) scored below the approved criterion (27%) in mathematical thinking. Based on this finding, we conclude that, during their academic preparation, these teachers received their teaching of mathematics by traditional lecture-based methods, without attention to the importance of developing thinking skills (Al-Fayez, 2016). Most of the students specializing in teaching have graduated from literary branch high schools, which lack materials that support mathematical thinking skills. Also, most of the students in this specialization performed poorly in high school exams. This can perhaps be attributed to the teachers" previous educational experiences that failed to develop their thinking skills (Stenger, 1999). We detected a significant difference between the performance of the teachers on the conceptual knowledge test and their performance on the procedural knowledge test; procedural knowledge is favored at the expense of conceptual knowledge. The teachers are not used to dealing with conceptual questions, whether while teaching students during the field training period or during their university studies (Kashim, 2016; Stenger, 1999).

We detected significant differences between the performance of the teachers on the conceptual knowledge test and their performance on the procedural knowledge test. This finding suggests that procedural knowledge is favored at the expense of conceptual knowledge during the teachers" university studies (Kashim, 2016).

This result suggests that the teachers learn mathematics as mere rules and procedures and apply them as algorithms to solve some problems, without understanding the concepts (Schneider et al. 2009; Kashim, 2016; Zuya, 2017).

Regarding the fifth question, we detected a positive relationship between the level of conceptual knowledge and mathematical thinking among Jordanian teachers (grades one to three) ($r = 0.323$, $p = 0.001$). This is likely because those teachers with good conceptual knowledge are able to understand mathematical ideas, realize the relationships and interconnection between different mathematical concepts. Thus, it is natural for them to be able to apply logical, deductive, and inductive reasoning and practice higher thinking skills (Darey et al., 2012).

We detected a significant, positive relationship between the level of procedural knowledge in the teaching of mathematics and mathematical thinking among Jordanian teachers (grades one to three) ($r = 0.676$, $p < 0.001$).The mathematical thinking skills are needed to be able to apply logical, deductive, and inductive reasoning, and practice higher thinking skills.

When we talk about conceptual knowledge and procedural knowledge, we talk about mathematical knowledge(Hill et al., 2005). Based on our findings, we recommend that study programs should be developed for colleges that can better prepare teachers; so that the teacher is a product of knowledge and not a preserver of it. Teachers should also have the option to specialize (e.g., as mathematics, science, or English language teachers).

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References

Baker, W., &Czarnocha, B. (2002). Written meta-cognition and procedural knowledge. Proceedings of the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level). University of Crete, Hersonissos, Crete.

Darey, H., Terzinha, N., Peter, B., &Christina, T. (2012). Individual differences in conceptual and procedural fraction understanding: The role of abilities and school experience.Journal of Experimental Child Psychology, 113(4), 469– 486.

Groth, R. E.,&Bergner, J. A. (2006). Preservice Elementary Teachers‟ Conceptual and Procedural Knowledge of Mean, Median, and Mode. Mathematical Thinking and Learning, 8(1), 37–63.

Halford, S. (1993**).Children's Understanding: The Development of Mental Models**. Hillsdale, NJ: Erlbaum.

Hill, H. C., Rowan, B., &Ball, D. L. (2005). Effects of teachers mathematical knowledge for teaching on studentachievement**. American Educational Research Journal**, 42(2), 371–406.

Kashim, R. M. (2016). The conceptual and procedural knowledge of rational numbers in primary school teachers. International Journal of Social, 10(3), 933–93.

Kilpatrick, J. (2020). History of Research in Mathematics Education. Encyclopedia of Mathematics Education. DOI: https://doi.org/10.1007/978-3-030-15789-0_71

National Council of Teacher of Mathematics NCTM. (2000). Principles and Standards for School Mathematics.**Reston.**

Rittle-Johnson, B. & Schneider, M. (2012). Developing conceptual and procedural knowledge of mathematics. In R. Cohen Kadosh& A. Dowker (Eds.), Oxford Handbook of Numerical Cognition. Oxford University Press.

Rittle-Johnson, B., Siegler, R., &Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. Journal of Educational Psychology, 93(2): 346–362.

Rodríguez-Martínez, J. A., González-Calero, J. A., &Sáez-López, J. M. (2020). Computational thinking and mathematics using Scratch: an experiment with sixth-grade students. Stories From the Field: Locating and Cultivating Computational Thinking in Spaces of Learning, 28, 316–327.

Schneider, M., Grabner, R. H., &Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: their interrelations in grades 5 and 6**.Journal of Educational Psychology,**101(2), 359–372.

Star, J. R. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36, 404–411.

Stenger, C. (1999). Characterization of University Students‟ Mathematical Thinking**. DAI, AAT 9949202,**University of Missouri- Kansas City.

Surif, J.(2012). Nor hasnizaibrahimmahanimokhtar. Conceptual and Procedural Knowledge in Problem Solving,56, 416–425.

Tchoshanov,M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics.Educational Studies in Mathematics, 76, 141–164.

Zuya, E. (2017). Prospective teachers‟ conceptual and procedural knowledge in mathematics: The case of algebra**.American Journal of Educational Research,**5(3), 310–315.

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