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Students' Conceptual Error and Procedural Error in Solving Algebraic Problems

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Abstract

In solving algebraic problems, there are two possible student answers, namely right or wrong. However, the correct answer is not necessarily through a process that is in accordance with the actual concept or what is called pseudo. Errors and pseudospheres made by students when solving algebraic problems can be grouped into conceptual errors and procedural errors. Conceptual errors and procedural errors are mistake that cannot be ignored in the learning of prospective mathematics teachers. Teachers need to identify these errors in order to provide corrective or corrective instructions. The purpose of this study was to identify and characterize the types of student errors in solving algebraic problems and to describe students' conceptual and procedural errors in solving algebraic problems. The design used for this research is a mixed method. There are two stages in this research. The first stage, identifying and characterizing the types of student errors in solving algebraic problems. The second stage, describes students' conceptual errors and procedural errors in solving algebraic problems. The subjects of this study were 92 students of the Mathematics Education Study Program at a university in South Sulawesi. The results of the study show that conceptual errors in algebra are caused by misconceptions about certain concepts, making equivalence between several concepts without regard to conditions, and ambiguity in interpreting mathematical symbols. Meanwhile, procedural errors are more errors at the completion stage due to the generalization of the rules.

Introduction

Algebra is one of the topics in school mathematics which is important to study (Erbilgin & Şahin, 2021; Jin & Wong, 2015; Lee et al., 2020). It is useful in science, engineering, and calculation. In algebra there is manipulation symbol, so that an expression can be transformed into another expression by certain rules without changing the meaning. This feature makes algebra a powerful tool for solving mathematical problems and even problems in everyday life. Despite the benefits, it is generally found that a child shows aversion to Algebra. Many argue that learning Algebra is more difficult than learning to count (Guler & Celik, 2021).

In solving algebraic problems, students are more likely to work procedurally without identifying the relational elements formed in the expression. One of the errors that are still often encountered is when students are asked to determine the solution of the equation $4x + 6 = 2(2x + 3)$.There are still students who have difficulties. Students don't know what to do with the loss of the xchanger which shows their understanding is still very shallow. Where they do not see that the object generated in the first step $4x + 6 = 4x + 6$ shows that the expression in the left field is equal to the expressei in the right field. Teacher must carefully predict the student's understanding, lest his understanding only be seen in routine matters.In learning, students seem to understand mathematical problems, as if indicating that they have a deep understanding of the material, especially algebra. When solving algebraic problems, there are only two possible answers of students, namely right or wrong. But the correct answer, not necessarily through the process that corresponds to the actual concept or the so-called pseudo. Pseudo means as if someone's answer is correct, but after searching, it turns out that what the person thinks does not correspond to the substance of the concept (Subanji & Nusantara, 2013). Mistakes and pseudo-made by students when solving algebraic problems can be grouped into conceptual errors and procedural errors.

Conceptual Error is an error caused by misconceptions or a wrong understanding of the principles and ideas underlying mathematical problems (for example, the relationship between numbers, characteristics, and properties of forms)(Lai, n.d.). Conceptual Error occurs because the student has misunderstood the underlying concept or has used the wrong logic (Chamundeswari, 2014). In solving problems, maybe all mathematical calculations are correct but if you understand a concept wrong, it can result in the use of the wrong procedur. Conceptual Error is the most difficult error to identify at first glance, it is also the most difficult error to recognize but it is the most important error to catch and correct.

Errors that occur in solving algebraic problems, not only conceptual errors but also procedural errors. Procedural error is an mistake that occurs because it does not apply the correct rules or algorithms (that is, formulas or step-by-step procedures for solving problems). As we know that procedural understanding is an understanding of the steps or procedures required in solving a problem. Procedural error refers to errors in the process of executing algorithmic procedures, which include operations, algorithmsa, placements, and incorrect stepsas well as missing steps in problem solving (Herholdt & Sapire, 2014; Mononen, 2022). The error in the settlement is unsystematic, the error is due to not being able to manipulate the resolution process (not being able to manipulate the settlement process due to the difficulty of how to animate to the next process), and the error of simplifyingis a part of the procedural error. Procedural error is the most common error so it is very easy to identify than conceptual error. Some studies that examine student's error include Oktaviani (2018) to analyze Students' Error in Doing Mathematics Problem on Proportion, Series (2019) research about Students' Error in Solving Mathematics Problem, (Mononen, 2022) research about Error analysis of students with mathematics learning difficulties in Tibet, Chauraya (2019) analyze In-Service Teachers' Perceptions and Interpretations of Students' Errors in Mathematics, etc. In this study, it examines Conceptual error and procedural error in algebra.

For mathematics education students who are prospective mathematics teachers, a deep understanding of mathematical concepts is very important. Thus, Conceptual error and procedural error are errors that cannot be ignored in the learning of prospective mathematics teachers. Teachers need to identify these errors to provide corrective or corrective instructions. Understanding and explaining these mistakes is important if teachers want to help students overcome some common mistakes in algebra. Furthermore, it is used as a basis for choosing the right and more challenging learning focus. In this step, teachers need to specifically address students' specific weaknesses. Based on this, the purpose of this study is to identify and characterize the types of student errors in solving algebra problems and describe student conceptual errors and procedural errors in solving algebra problems.

Method

The study was conducted in two stages. *The first stage*, identifying and characterizing the types of mistakes of the students in solving algebraic problems. *The second stage,* describes the *conceptual* error and *procedural error* of students in solving algebraic problems. In the first stage using quantitative descriptive data analysis and the second stage using aqualitative descriptive analysis, karena the design used can be *mixed method.* Quantitative exploration is needed to illustrate how much students make in solving algebraic problems. After finding the distribution of errors proceed with qualitative exploration. In this case, it is to describe *the conceptual*error and*procedural error* of students in solving algebraic problems.

The subjects of this study were 92 students of the Mathematics Education Study Programat one of the universities in South Sulawesi. Research instruments in quantitative design are the main instruments and tracking instruments. The main instrument contains 5 statements that require students to justify right or wrong accompanied by reasons. The tracking instrument is in the form of 5 statements that have been accompanied by several reasons, the subject is asked to answer agree or tidisagree with the reason. Tracking instruments are intended to clarify or capture the possibilities that students think about related to mathematical concepts. The tracking instrument is also intended to trace the construction in the student's mind, which will then make it easier to catch the student's mistakes in solving the problem.

Research instruments in qualitative design consist of main instruments and supporting instruments. The main instrument is the researcher, while the supporting instrument is the tracking instrument that has been compiled in a qualitative design. In quantitative design, .Astudentawaban is grouped in three possibilities: true, pseudo, clarification and false. "Benar" occurs if the student answers correctly and is able to give a reason correctly. The "pseudo" answer occurs if the student answers the question correctly but the reasons put forward lack important elements of the true conceptual and analytical thought process. "clarification" if the student answers correctly but does not give a reason. A "wrong" answer occurs if a student's answer is wrong and the reasons written in favor of his mistake.

Data analysis in this study was carried out in two forms, namely quantitative data analysis and qualitative data analysis. Quantitative data analysis is used by conducting descriptive analysis. The characteristics of student errors in solving problems are described using percentages.
Qualitative data analysis is carried out using the following steps as in diagram 2.1.

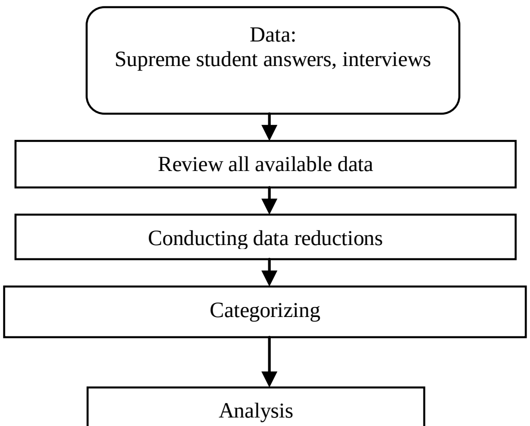

Diagram 1 Data Analysis Procedure

Results and Discussion

A. the types of mistakes of the students in solving algebraic problems

Student mistkes in solving problemsin algebra are divided into four forms: the operation of algebraic forms, ranks, solutions of equations, and solutions of inequality. The following table is the result of grouping student answers by main instrument and tracking instrument.

Table 1. Students Group based on the Result on Main and Tracker Instrument

Statement	True			False
	True	Pseudo	Without Clarification	
$-(xy) = (-x)(-y)$	72	0	9	11
$(a+b)^{-1} = a^{-1} + b^{-1}$	40	0	20	32
$x + 5 - \sqrt{x+5} = 6$ The value that satisfies the equation is or $xx = 4x = -1$	19	0	12	61
$x^2 = 9$, The value that satisfies the equation is $x = 3$ atau $x = -3$	23	69	0	0
$-4x < 12$ The value of x that meets the answer is $x < -3$	30	0	20	46

Based on Table 1, there are still 11 (11.96 %) students who state that the statement is true $-(xy) = (-x)(-y)$. The number of students who statedtrue statements $(a+b)^{-1} = a^{-1} + b^{-1}$ was 32 (34.8%). The mistakes that students make are related to the procedure. This mistake comes from a wrong generalization of distributive properties. Here is the answer of the student group called the first Subject group (S1) to the first statement

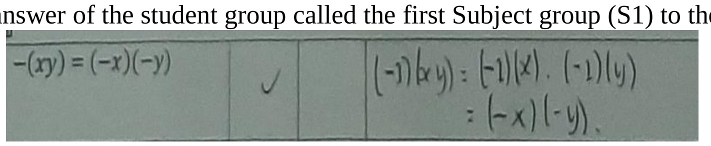

Figure 1. S1's Answer to the First Question

S1 states that the statement is true. The way S1 does this is by separating variables and xy then using distributive properties, i.e. multiplying (-1) separately to x and y. S1's answer is consistent when given a tracker question. Here is the student group's incorrect answer to the statement $(a+b)^{-1} = a^{-1} + b^{-1}$. This group of students is called subject group 2 (S2).

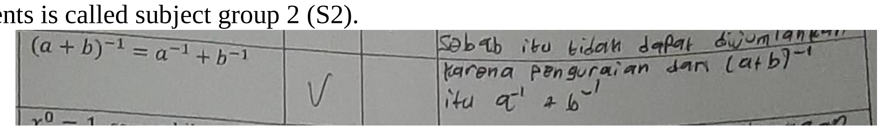

Figure 2. S2's Answer to Second Statement

Students' understanding begins $(a+b)^1 = a + b$ with and applies it to $(a+b)^{-1}$ be acquired $a^{-1} + b^{-1}$ based on distributive nature. The mistakes students make are procedural errors. This happens because students only remember the distributive nature vaguely so they cannot apply to the appropriate problem. Another mistake made by a group of students called subject group 3 (S3) is $(a+b)^{-1} = \frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$. Fractions $\frac{1}{a} = a^{-1}$ and $\frac{1}{b} = b^{-1}$.

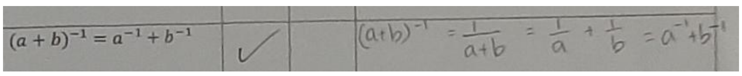

Figure 3. S3's Answer to Second Statement

The student group is on fractions operations. This group of students reveals that $\frac{1}{a+b}$ is equivalent to $\frac{1}{a}+\frac{1}{b}$. Furthermore, there are still many students who are wrong in completing the third statement. The total is 61 students or 66.3%. This group of students is called subject group 4 (S4). Procedurally, the value of x obtained is $x = 4$ and $x = -1$. But it turns out that $x = -1$ is not the solution of the equation $x + 5 - \sqrt{x+5} = 6$. This shows that students can work through each step carefully and correctly but still get the wrong answer.

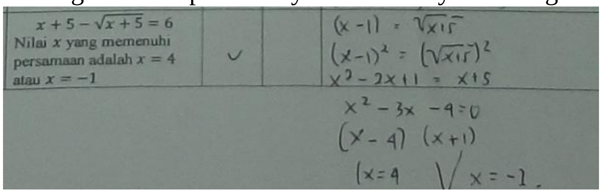

Figure 4. S4's Answer to Statement Three

S4 applies the fact of $a = b$ so that $a^2 = b^2$ then the equation becomes $\left(\sqrt{x+5}\right)^2 = (x-1)^2$ obtained $x + 5 = x^2 - 2x + 1$. In this process, the student group understands that the rank of two and the square root are equivalent so that the withdrawal of the root can be done by raising the rank of two. The equation $x + 5 - \sqrt{x+5} = 6$ (*equation 1*) is added and on both sides of it, further multiplied by (-1), is obtained $\sqrt{x+5} = x - 1$ (*equation 2*). What students do not understand is that the squaring of equation 2 into $x + 5 = x^2 - 2x + 1$ (*equation 3*) is justifiable for a certain x value. Both sides of *equation* 3 added $(-x)$ and -5 obtained (*equation 4*) are also justified for a given value of $0 = x^2 - 3x - 4x$. That is, each solution in equation 2 is a solution of equation 4. In the case of sets, we can say that the set of solutions from *equation 2* is part of the set of solutions from *equation 4*.

A total of 69 (75%) students who are *pseudo* when working on statements"$x^2 = 9$, the value that meets the equation is or $x = 3$ or $x = -3$". This group of students is called subject group 5 (S5). S5 answers $x^2 = 9$ then $= \sqrt{9}$, so the solution is $x = 3$ or $x = -3$.

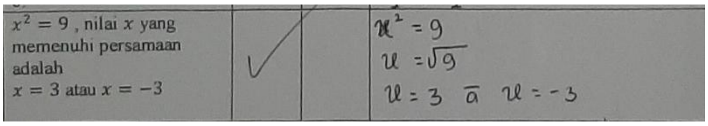

Figure 5. S5's Answer to the Fourth Statement

All students correctly answer the statement"$x^2 = 9$. The value that meets the equation is $x = 3\ or\ x = -3$". However, there is a group of students, they are S5 who answer $x^2 = 9$ equivalent to $= \sqrt{9}$, so the solution is $x = 3$ or $x = -3$. This happens because they do not understand about the square root, which is based on the definition $\sqrt{x^2} = |x|$ of . So $\sqrt{9}$ it can be interpreted by the nonnegarif square root of 9. Here is another answer from the Subject 6 (S6) group to the statement "$x^2 = 9$".

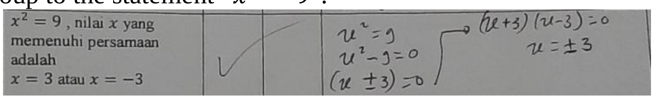

Figure 6. S 6's Answer to the Fourth Statement

Another mistake made by students related to quadratic equations is the meaning of symbols as less and $\pm$ plus operating symbols or positive and negative signs of a number.

In the statement $-4x < 12$, there are 46 (50%) students who are wrong. This group of students or subject group 7 (S7) applies the "normal" rule in solving equations even when they are dealing with inequality.

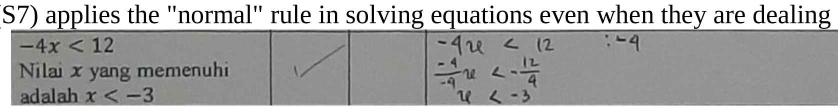

Figure 7. Answer S7 to the Fourth Statement

S7 confirms that the interval that can make the inequality applied is $x < -3$ without retesting whether the solution is true or not. In determining the solution interval, students multiplythe same negative number $-\frac{1}{4}$ to both fields of inequality but do not reverse the symbol "less than (<)" to "more than (>). Students are said to have an understanding of mathematical concepts if they are able to interpret concepts relationally (Jin & Wong, 2015; Nor Hasnida & Effandi, 2011). In the concept of inequality, the use of larger (>) and smaller (<) symbols needs to be considered especially when it comes to the nature of multiplication in inequality. It is very important to reverse the direction of the symbol of inequality when multiplying by a negative number so that a new order is obtained equal to the original.

B. Students' *Conceptual Error and Procedural Error* in constructing concepts and solving Soal Algebra

In constructing concepts and solving questions, there are two possible answers from students, namely true or false. There are three possibilities for a correct answer, namely a correct answer with the right reason, a correct answer with a less logical or false reason called pseudo, and a correct answer without a reason. Pseudo and incorrect answers are caused by two reasons, namely the existence of a conceptual error or procedural error.

Conceptual error is related to a lack of understanding. Mistakes made by students when constructing and solving algebraic problems occurs because they do not have a complete understanding of mathematical concepts (Mononen, 2022).

Table 2. Description of the Conceptual Error of Pseudo and Incorrect Students in Solving the Problem

Conceptual Error	Statement	Mistake
Misconceptions of square root and squared: Students point out that square root and squared are equivalent operations. Root withdrawal can be done by means of squared.	$x+5-\sqrt{x+5}=6$ The value of x that satisfies the equation is $x=4$	Students stated $x=-1$ as one of the solutions to the equation.
Misconceptions of quadratic equations and quadratic roots	$x^2=9$, the value that satisfies the equation is $x=3$ or $x=-3$	Students answer that $\sqrt{9}$ are 3 and -3.
Ambiguous mathematical meaning: meaning of symbol $\pm$		Ambiguous in interpreting symbols $\pm$ as less and plus operations or negative and positive signs of a number.

In answering the third statement, the student group (S4) procedurally obtained the solution of the equation correctly. However, if investigating by substituting the set of solutions to the equation, then one member of the set is obtained that is not the solution of the equation. The student group (S5 and S6) answered correctly fourth. But in the process, there is a conceptual error, which is a mistake made by students because they do not have a complete understanding of the concept of square root and rank two as well as ambivalence in interpreting mathematical symbols such as $\pm$. Conceptual Error may not look like a procedural error, but it does happen because students do not fully understand certain mathematical concepts. Conceptual Error is a very serious mistake. Students experience Conceptual Error due to misconceptions. Misconceptions seem to be a common phenomenon among students in mathematics learning. Misconception is defined as "the conception of students that produces systematic patterns of error"(Ainiyah et al., 2019; Engelbrecht et al., 2005; Kshetree et al., 2021; Mills, 2016). This idea suggests that misconceptions are not easy to see, but are manifested through patterns of errors observed in student work. Mathematical misconceptions seem to be related to the inaccuracy of ideas that students develop in mathematics. Such misconceptions may be rooted in the early knowledge of college students (Agustyaningrum et al., 2018; Barbieri et al., 2019; Herholdt & Sapire, 2014; Krajcevski, 2019; Schnepper & McCoy, 2013). Students assume that what they are doing is right or are not sure what they are doing. Errors can occur due to the inability or lack of awareness to check the answers given, such as when answering the statement ", $x+5-\sqrt{x+5}=6\,x$ that satisfies the equation is or "$x=4\ or\ x=-1$. Students only perform procedural stages but do not double-check whether the solutions obtained can meet the equations or not. Persistent misconceptions can interfere the student's ability to understand mathematical concepts and can lead to frequent repetition of mistakes. Such errors can lead to low performance, causing anxiety towards the subject leading to a negative attitude and a bad mathematical image.

Procedural knowledge is the understanding of what steps or procedures are required to solve a problem. It means that*Procedural Error* is related to an error about the settlement procedure. The following tableis a *procedural error* in constructing and solving algebraic problems.

Table 3. The Procedural Error of Students Who are Wrong in Constructing and Solving Problems

Procedural Error	Statement	Mistake
The failure that occurs because studentsonly remember the distributive nature vaguely so that	$-(xy)=(-x)(-y)$	Students multiply (-1) by x and y consecutively.
	$(a+b)^{-1}$	Students distribute the squared of -1

they cannot apply to the corresponding problem.	$= a^{-1} + b^{-1}$	to a and b.
Failure in fractional summation operation		Students perform an add operation on two fractions by adding both fraction denominators $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$
Failure to multiply the same negative number to both fields of inequality: Students multiply the same negative number to both fields of inequality but do not reverse the symbol "less than (<).	$-4x < 12$ The value of x that meets is $x < -3$	Students multiply both sides of the inequality by numbers by $-\frac{1}{4}$without reversing the < mark to >.

Procedural error occurs because of the lack of basic knowledge in mathematical operations. This results in students solving the problem with the wrong step. Procedural errors are errors due to incorrect performance of steps in mathematical processes, for example failures due to incorrect use of the mathematics rules. The procedural errors made by students occurred because they only vaguely remembered the distributive properties, so they could not apply them to the appropriate problem. In this case, it happened Misgeneralization, students made mistakes by generalizing an existing concept (Andriani et al., 2021).

Conclusion

There are two answers for students in solving algebraic problems, namely correct or incorrect. Unfortunately, the correct answer does not necessarily go through the correct or pseudo-stages. That is, in the process of solving it does not correspond to the substance of the concept. The mistakes and pseudothat made by the students are caused by their superficial and false understanding. These errors are grouped in conceptual errors and procedural errors. Conceptual error in algebra is caused by misconceptions about certain concepts, making equality between several concepts regardless of conditions, and ambivalence in interpreting mathematical symbols. While the procedural error is more of an error at the completion stage due to the generalization of the rule or properties that are less preces. Conceptual errors and procedural errors are common and serious mistakes for students as prospective teachers, so it is necessary to minimize these errors. Innovative methods and techniques need to be adopted to make algebraic learning effective. Students should be given a clear idea of each stage in Algebra and each mathematical operation should be taught logically in steps. Expanding the practice is an early stage that will definitely add depth of understanding.

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